# The QED lowest-order radiative corrections to the two polarized identical fermion scattering

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**Abstract.** The explicit exact expressions of QED lowest-order radiative corrections to the two polarized identical fermion scattering are presented in covariant form. Polarization effects are treated in detail. The infrared divergence from the real photon emission is extracted by covariant approach. Some numerical results for Møller scattering are given.

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## 1. Introduction

The two identical fermion scattering has been investigated since the arising of *Quantum Electrodinamics*. In the early 1930s Møller [1] calculated the unpolarized cross section of this process at the Born level. The lowest-order radiative corrections (RC) to the two identical fermion scattering were first calculated by Redhead [2], Polovin [3, 4] and Tsai [5] for the unpolarized cross section. DeRaad [6, 7], using the approaches proposed by Mo and Tsai [8] and Gastmans and collaborators [9], carried out a calculation of RC taking into account particle polarization. However these results are set-up dependent (the unphysical parameter $\Delta E$ appears in the results) and the contribution of "hard photons" has been neglected. In [10] an exhaustive analysis of RC is presented but the results are also set-up dependent. In [11] Monte Carlo method was used for calculation of RC at SLAC set-up. Some separate contributions were widely discussed in [12, 13, 14].

Thus, we can conclude that the results of calculations of RC to the two polarized identical fermion scattering have been presented in non-covariant form, they were oriented mainly, to the study of scattering in the center of mass system and an exhaustive analysis of polarization effects has been not performed. The aim of this paper is to present a complete QED lowest-order calculation of RC to cross section and other observables in the process of the two polarized identical fermion scattering in covariant form. We propose formulae, which can be used both for numerical analysis of fixed target experiments and collider ones. A detailed analysis of polarization effects is presented. Since we consider the scattering of any two identical fermions it is desirable to have exact formulae, without approximations such as ultrarelativistic (URA) or leading log (LLA). One of the main applications of proposed results is the study of RC to polarized Møller scattering [1] and, in particular, to Møller polarimeter.

It is known that two identical fermion scattering is characterized by the presence of t- and u- channel diagrams and their interference. That fact makes the calculation of cross section more complicated than for two non-identical fermion scattering (e.g. $\mu^- e^-$), where only the contribution of one of these channels have to be considered and interference vanishes. The Feynman graphs, which contribute to the Born cross section of the considered process are shown on figure 1. The full set of Feynman graphs which is necessary to calculate the QED lowest-order RC is presented on figure 2.

The observed cross section is given by

$$\begin{aligned}\sigma_{obs} \sim \quad & |M_a + M_b|^2 + 2Re(M_a + M_b) \sum_{i,i'=1}^{5} (M_i + M_{i'})^+ + \\ & + \sum_{i,i'=6}^{9} |(M_i + M_{i'})|^2.\end{aligned} \tag{1}$$

Here $M_{a(b)}$ are the matrix elements of Born (or one-photon exchange) contribution. $M_{i(i')}$ are the matrix elements of radiative processes. In order to calculate exactly the QED lowest-order RC to the two polarized identical fermion scattering, the method offered in ref.[15] is used (for more details see also [16]-[22]).

This paper is organized as follows. In section 2 the kinematics for elastic process (Born and virtual photon(V) contributions) and inelastic one (real photon(R) contribution) is presented. Section 3 is devoted to the presentation of explicit formulae for V- and R-contributions. We conclude with an illustration and short discussion of obtained results in section 5.

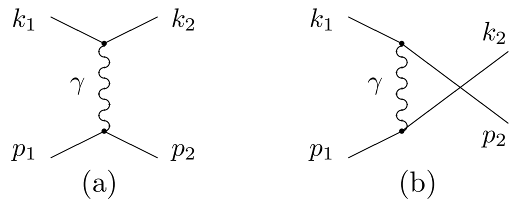


**Figure 1.** Feynman diagrams contributing to the Born cross section. t-channel (a) and u-channel (b).

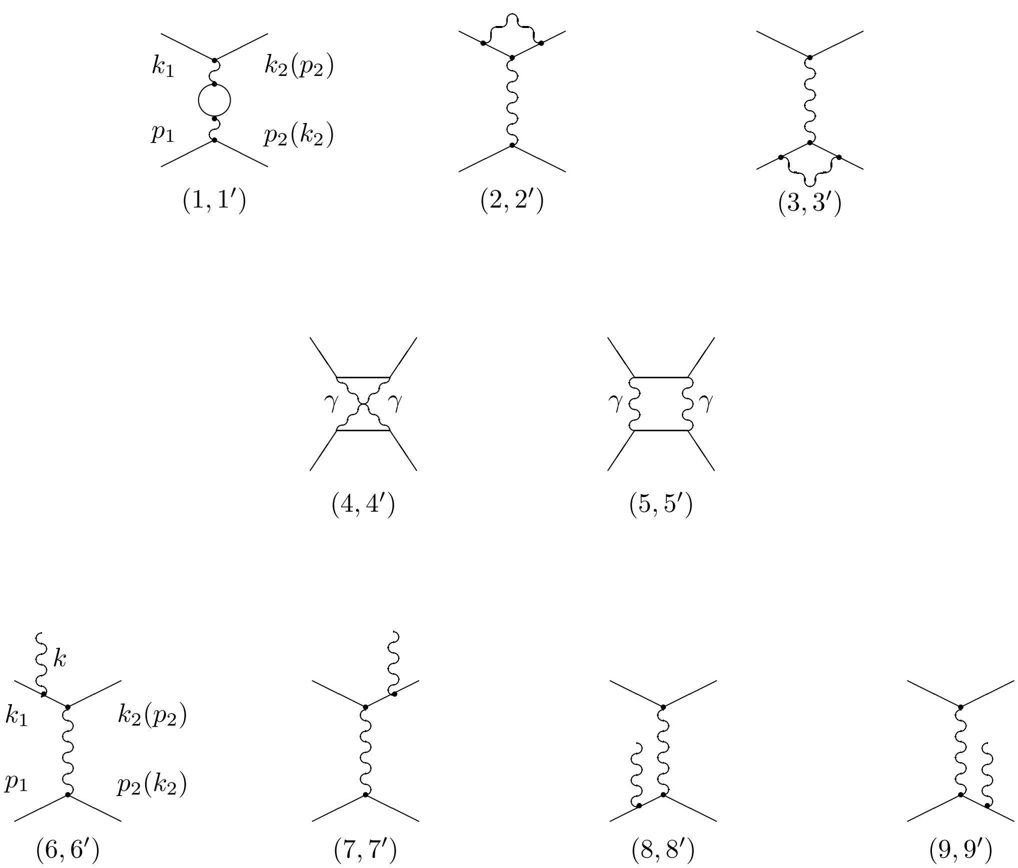


**Figure 2.** Feynman diagrams contributing to the QED lowest-order radiative corrections to the two identical fermion scattering.

## 2. Kinematics and phase space

We consider both the elastic scattering

$$f_1(k_1,\xi)+f_2(p_1,\eta)\to f_3(k_2)+f_4(p_2) \tag{2}$$

and the bremsstrahlung process

$$f_1(k_1,\xi)+f_2(p_1,\eta)\to f_3(k_2)+f_4(p_2)+\gamma(k) \tag{3}$$

where $k_1$,$p_1$($k_2$,$p_2$) are four momenta of initial(final) fermions and $k$ is the four momenta of a real photon. $\xi$, $\eta$ are the beam and target polarization vectors.
In the case of longitudinal (see [21]) polarization

$$\begin{aligned}\xi = \ \ \xi_l &= \frac{1}{\sqrt{\lambda_s}}\Big(\frac{S}{m}k_1-2mp_1\Big)\\ \eta=\eta_l &= \frac{1}{\sqrt{\lambda_s}}\Big(2mk_1-\frac{S}{m}p_1\Big)\end{aligned} \tag{4}$$

where $S=2p_1.k_1$, $\lambda_s=S^2-4m^4$, and $m$ is the fermion mass. If the beam and target are polarized transversally

$$\begin{aligned}\xi = \ \ \xi_t &= \frac{1}{\mathcal{K}\sqrt{\lambda_s}}\Big((-SX_o+2m^2Q^2_m)k_1+\lambda_s k_2-(SQ^2+2m^2S^o_x)p_1\Big)\\ \eta=\eta_t &= \xi_t.\end{aligned} \tag{5}$$

Here $Q^2=-(k_1-k_2)^2$, $Q^2_m=Q^2+2m^2$, $X_o=2p_1.k_2$, $S^o_x=S-X_o$ and $\mathcal{K}=(Q^2(SX_o-2m^2Q^2_m))^{\frac{1}{2}}$. Finally, if the beam and target are normal‡ polarized

$$\begin{aligned}\xi^\mu = \ \ \xi^\mu_\perp &= \frac{1}{\mathcal{K}}\epsilon_{\mu\lambda\gamma\theta}k_1^\lambda k_2^\gamma p_1^\theta\\ \eta^\beta=\eta^\beta_\perp &= \frac{1}{\mathcal{K}}\epsilon_{\beta\lambda\gamma\theta}k_1^\lambda k_2^\gamma p_1^\theta\end{aligned} \tag{6}$$

$$\eta^2=\eta^2_\perp=\xi^2_\perp=-1 \tag{7}$$

where $\epsilon_{\mu\lambda\gamma\theta}$ is the Levi-Civita's tensor.
The cross section of (3) depends on five kinematical variables. We use the independent set:

$$S,\ \ y=\frac{S^o_x}{S},\ \ v=2p_2.k,\ \ t=-(p_2-p_1-k)^2,\ \ z_2=2k_2.k.$$

Using the parametrizations (4), (5) and (6), a general expression of the Born cross section of the two polarized identical fermion scattering can be presented in the form§:

$$\begin{aligned}\sigma_o = \ \ &\frac{4\pi\alpha^2 S}{\lambda_s}\Big[\Big(\frac{U_1}{Q^4}+\frac{U_2}{Q^4_1}+\frac{U_3}{Q^2Q^2_1}\Big)+\\ &+\sum_B\sum_T P_BP_T\Big(\frac{P_{1BT}}{Q^4}+\frac{P_{2BT}}{Q^4_1}+\frac{P_{3BT}}{Q^2Q^2_1}\Big)\Big].\end{aligned} \tag{8}$$

‡ normal it means perpendicular to the scattering plane.
§ Here and below $\sigma=\frac{d\sigma}{dy}$.

In (8) indexes $B$(beam) and $T$(target) mean $B$=$T$=$l$(long.), $t$(transv.), $\perp$(normal). The quantities $U_i$ and $P_{iBT}$ are given in explicit form in **appendix A**, $Q_1^2 = X_o - 2m^2$ and $P_{B(T)}$ are the polarization degree of beam(target) fermions.

## 3. Exact formulae for the lowest-order radiative corrections

The cross section of analysed process in order $\alpha^3$ is given by

$$\sigma = \sum_k(\sum_j \sigma_k^{u,j} + P_B P_T \sum_j \sigma_k^{p,j}) + \sum_h(\sum_n \sigma_h^{u,n} + P_B P_T \sum_n \sigma_h^{p,n})). \tag{9}$$

Here $j$ runs over contributions of vacuum polarization, vertex diagrams and bremsstrahlung. The index $k$ points out sum over the contributions of t-($l$), u-($e$) channel diagrams and their interference($i$). The index $n$ denotes sum over anomalous magnetic moment and two-photon exchange. For these last corrections the contribution of the interference of t- and u- channel diagrams has been divided into two parts. For this reason we have separated these corrections from the other ones and use the index $h$ instead of $k$ (see bellow formulas (20) and (21)). The index $u(p)$ is used to separate the contribution of unpolarized(polarized) parts.

### *3.1. V-contribution*

We call "factorized RC" the sum of all $\delta$ which are looked like that $\sigma = \delta\sigma_o$. It is very convenient, for numerical analysis, to collect together all contributions with factorized corrections

$$\sigma_{fact}^{u(p)} = \frac{\alpha}{\pi}\sum_k\sum_f \delta_f^k \sigma_o^{u(p),k}. \tag{10}$$

In (10) $\sum_k \sigma_o^{u(p),k}$ are the unpolarized(polarized) parts of the Born cross section (8). The index $f$ means: $f$=$vp$, $vert$, $L$, $\lambda$, $s$ ,$K$. It is evident from (10), that corrections $\delta_f^k$ do not depend on polarization.

The quantity $\delta_{vp}$ is the well known correction due to vacuum polarization (see figure 2(1,1′))

$$\delta_{vp}^l = \sum_{i=e,\mu,\tau,...} q_i^2\left[\frac{2}{3}\left(Q^2 + 2m_i^2\right)L_{m_i} - \frac{10}{9} + \frac{8m_i^2}{3Q^2}\left(1 - 2m_i^2 L_{m_i}\right)\right] \tag{11}$$

where

$$L_m = \frac{1}{\sqrt{\lambda_m}}\ln\frac{\sqrt{\lambda_m}+Q^2}{\sqrt{\lambda_m}-Q^2} \qquad \lambda_m = Q^2(Q^2+4m^2)$$

and

$$\delta_{vp}^i = \delta_{vp}^l + \delta_{vp}^e \tag{12}$$

where $\delta_{vp}^e$=$\delta_{vp}^l(Q^2 \to Q_1^2)$. In (11) $q_i$ is the charge of leptons or quarks. The quantity $\delta_{vert}$ is the correction due to convergent part of vertex diagrams (see figure 2(2,2′; 3,3′))

$$\delta_{vert}^l = 2\left(\frac{3}{2}Q^2 + 4m^2\right)L_m - 4 \tag{13}$$

$$\delta^i_{vert} = \delta^l_{vert} + \delta^e_{vert} \tag{14}$$

where $\delta^e_{vert}$=$\delta^l_{vert}(Q^2 \to Q_1^2)$. The correction $\delta_L$, obtained from the sum of finite terms derived from infrared divergent part of vertex and two-photon exchange diagrams, is:

$$\delta^l_L = 2[L(p_1,k_1) - L(k_1,k_2) - L(p_1,k_2)] \tag{15}$$

$$\delta^l_L = \delta^e_L = \delta^i_L. \tag{16}$$

The explicit expressions of $L(p_1,k_1)$, $L(k_1,k_2)$ and $L(p_1,k_2)$ are given in [15] (see (66)). The correction $\delta_\lambda$, coming from the sum of infrared divergent parts of V- and R-contributions, is:

$$\delta^l_\lambda = J(Q^2,0)\ln\frac{v_{max}}{2m^2} \tag{17}$$

$$\delta^l_\lambda = \delta^e_\lambda = \delta^i_\lambda. \tag{18}$$

Here

$$J(Q^2,0) = 2[2(Q^2_m L_m - 1) + X_o L_{X_o} - S L_S]$$

$$L_S = \frac{1}{\sqrt{\lambda_s}}\ln\frac{S+\sqrt{\lambda_s}}{S-\sqrt{\lambda_s}}$$

$$L_{X_o} = L_S(S \to -X_o), \quad \lambda_{x_o} = \lambda_s(S \to -X_o)$$

$$v_{max} = \frac{2Q^2 d_s(Q^2_{max} - Q^2)}{Q^2 d_s + \sqrt{\lambda_m\lambda_s}}, \; Q^2_{max} = \frac{\lambda_s}{d_s}, \; d_s = S + 2m^2.$$

The quantity $\delta_s$ is the finite part of the "soft-photon" contribution:

$$\delta^l_s = \delta^e_s = \delta^i_s. \tag{19}$$

The explicit expression for $\delta_s$ was given in [15] (see (56)). The correction $\delta_K$ follows from diagrams of two-photon exchange. Its explicit form was presented first in [19] (see (A45)).

The contribution of the anomalous magnetic moment (see figure 2(2,2′; 3,3′)) can be expressed as

$$\begin{aligned}\sigma_{amm} = \; & \frac{8\pi\alpha^2 m^2 S}{\lambda_s}\frac{\alpha}{\pi}\Big[\Big(\frac{A_1}{Q^2} + \frac{A_2}{Q^2Q_1^2}\Big)L_m + \Big(\frac{A_3}{Q_1^4} + \frac{A_4}{Q^2Q_1^2}\Big)L_{mex} + \\ & + \sum_B\sum_T P_B P_T\Big(\Big(\frac{A_{5BT}}{Q^2} + \frac{A_{6BT}}{Q_1^2}\Big)L_m + \\ & + \Big(\frac{A_{7BT}}{Q_1^4} + \frac{A_{8BT}}{Q^2Q_1^2}\Big)L_{mex}\Big)\Big].\end{aligned} \tag{20}$$

Here $\lambda_{mex} = \lambda_m(Q^2 \to Q_1^2)$, $L_{mex} = L_m(Q^2 \to Q_1^2)$. The explicit form of the quantities A is presented in **appendix C**.

The contribution of the two-photon exchange diagrams (see figure 2(4,4′; 5,5′)) is given by

$$\begin{aligned}\sigma_{2\gamma} =\ & \frac{\alpha}{\pi}\Big(\delta^{l,u}_{2\gamma}\sigma^{l,u}_{o} + \delta^{e,u}_{2\gamma}\sigma^{e,u}_{o} + (\delta^{i1,u}_{2\gamma} + \delta^{i2,u}_{2\gamma})\sigma^{i,u}_{o}\Big)+ \\ & + \sum_B\sum_T P_B P_T \frac{\alpha}{\pi}\Big(\delta^{l,p}_{2\gamma BT}\sigma^{l,p}_{oBT} + \delta^{e,p}_{2\gamma BT}\sigma^{e,p}_{oBT} + \\ & +(\delta^{i1,p}_{2\gamma BT} + \delta^{i2,p}_{2\gamma BT})\sigma^{i,p}_{oBT}\Big).\end{aligned} \tag{21}$$

The explicit formulae for the corrections $\delta^{h,u(p)}_{2\gamma(BT)}$ are given in the **appendix D**.

### *3.2. R-contribution*

In order to extract the infrared divergence, which appears when we integrate over the real photon phase space, into a separate term we make, according to [15], an identity transformation‖

$$\sigma_R = \sigma_R - \sigma_{IR} + \sigma_{IR} = \sigma^F_R + \sigma_{IR} \tag{22}$$

where $\sigma_{IR}$ is infrared divergent and $\sigma^F_R$ (see **appendix E**) is finite when $k \to 0$.

$$\sigma_{IR} = \frac{\alpha}{\pi}\delta_{soft}\sum_k \sigma^k_o + \sigma^H \tag{23}$$

For the “soft” part we have

$$\delta_{soft} = \frac{1}{\pi}\int\limits_0^{\bar v} dv \int \frac{d^{n-1}k}{(2\pi\mu)^{n-1}k_0} F_{IR}\delta((\Lambda-k)^2-m^2) \tag{24}$$

where $\mu$ is an arbitrary parameter of mass dimension, $n$ is the dimension of space, $\Lambda = p_1 + k_1 - k_2$ and

$$F_{IR} = -\frac{m^2}{z_1^2} - \frac{m^2}{z_2^2} + \frac{Q^2_m}{z_1 z_2} + \frac{X}{u z_2} + \frac{X}{u z_1} - \frac{S}{v z_2} - \frac{S}{v z_1} - \frac{m^2}{u^2} - \frac{m^2}{v^2} + \frac{Q^2_m}{uv}.$$

Here $z_2 = 2k_2.k$, $u = 2p_1.k$ and $X = S - Q^2 - v = S(1-y) - v$.
The “hard” part is presented as

$$\sigma^H = \frac{\alpha}{\pi}\bar\delta\sum_k \sigma^k_o + \sigma^H_1. \tag{25}$$

In this expression the first term is infrared divergent and the second one is free of infrared divergence and is given explicitly by

$$\begin{aligned}\sigma^H_1 =\ & \frac{4\pi\alpha^2 S}{\lambda_s}\frac{\alpha}{\pi}\int\limits_0^{v_{max}} dv\Big[\Big(\frac{-X_o}{2Q^4} + \frac{m^2}{Q_1^4} + \frac{m^2}{Q^2Q_1^2}\Big)J(Q^2,v) \\ & +\Big(\frac{U_1}{Q^4} + \frac{U_2}{Q_1^4} + \frac{U_3}{Q^2Q_1^2}\Big)\Delta J(Q^2,v) + \sum_B\sum_T P_B P_T T_{BT}\Big].\end{aligned} \tag{26}$$

‖ For details about the method used for cancellation of infrared divergence see [15]

Here

$$
\begin{aligned}
T_{ll} =& -\Big(\frac{S}{2Q^4}+\frac{1}{Q^2}\Big)J(Q^2,v)+\Big(\frac{P_{1ll}}{Q^4}+\frac{P_{2ll}}{Q_1^4}+\frac{P_{3ll}}{Q^2Q_1^2}\Big)\Delta J(Q^2,v)\\
T_{tt} =& \Big(\frac{P_{1tt}}{Q^4}+\frac{P_{2tt}}{Q_1^4}+\frac{P_{3tt}}{Q^2Q_1^2}\Big)\Delta J(Q^2,v)\\
T_{\perp\perp} =& \Big(\frac{2Q_1^2-v}{2Q^2Q_1^2}\Big)J(Q^2,v)+\Big(\frac{P_{1\perp\perp}}{Q^4}+\frac{P_{2\perp\perp}}{Q_1^4}+\frac{P_{3\perp\perp}}{Q^2Q_1^2}\Big)\Delta J(Q^2,v)\\
T_{lt} =& -\Big(\frac{\mathcal{K}}{2mQ^2Q_1^2}\Big)J(Q^2,v)+\Big(\frac{P_{1tt}}{Q^4}+\frac{P_{2tt}}{Q_1^4}+\frac{P_{3tt}}{Q^2Q_1^2}\Big)\Delta J(Q^2,v)\\
T_{tl} =& T_{lt} \qquad T_{l\perp}=T_{\perp l}=T_{t\perp}=T_{\perp t}=0\\
J(Q^2,v) =&\ \ 2(Q_m^2L_m-1)+X(L_X+L_{AX})-\\
&-S(L_S+L_{AS})+2(Q_m^2L_Y-1)+\frac{v}{\tau}\\
\tau=&m^2+v\\
\Delta J(Q^2,v) =& \frac{1}{v}\Big[J(Q^2,v)-J(Q^2,0)\Big].
\end{aligned}
$$

The explicit expressions for $L_X$, $L_{AX}$, $L_{AS}$ and $L_Y$ may be found in [15] (formulae (32), (33), (34) and (38) respectively).

At the last we present a scheme for approximate consideration of the "multi-soft-photon" emission. It is achieved by means of the so-called exponentiation procedure. The correction due to the consideration of "soft-photons" can be given by

$$\delta^l_{inf}=2\Big[\Big(\ln\frac{Q^2}{m^2}-1\Big)\ln\frac{v^2_{max}}{SX_o}+\ln\frac{X_o}{m^2}\ln\frac{v^2_{max}}{SQ^2}-\ln\frac{S}{m^2}\ln\frac{v^2_{max}}{X_oQ^2}\Big] \tag{27}$$

$$\delta^l_{inf}=\delta^e_{inf}=\delta^i_{inf}. \tag{28}$$

(See for more details [8], [23]-[26]). This correction modifies the expression of cross section (9) in the next form:

$$
\begin{aligned}
\sigma+\sum_k\sigma_o^k \rightarrow\ \ & \exp\Big(\frac{\alpha}{\pi}\sum_k\delta^k_{inf}\Big)\Big[\sum_k\Big(1+\frac{\alpha}{\pi}\sum_f\delta^k_f-\frac{\alpha}{\pi}\delta^k_{inf}\Big)\sigma_o^k+\\
&+\sum_k\Big(\sigma_R^{k,F}+\sigma_1^{k,H}\Big)+\sum_h\Big(\sigma^h_{2\gamma}+\sigma^h_{amm}\Big)\Big].
\end{aligned} \tag{29}
$$

In this formula the Born contribution has been included in $\sigma$ (9). The sums over $k$, $h$ and $f$ have the same sense as in (9) and (10).

## 4. Conclusions

The QED lowest-order RC to the two identical polarized fermion scattering have been calculated using a covariant approach. The contributions of "hard" and "soft" photons as well as two-photon exchange have been considered exactly. The ultrarelativistic approximation is used to consider the "multi-soft-photon" contribution. Obtained results are free of set-up dependent parameters (e.g. $\Delta E$). The analytical results have

been obtained using the system of analytical calculations REDUCE [27] and are in agreement with cross-symmetry requirements.

We use polarized Møller (electron-electron) scattering at fixed target experiment to illustrate (figure 3) how the formulae, given above, work. Numerical calculation was perfomed using the FORTRAN code MØLLERAD without any experimental cuts and restrictions.

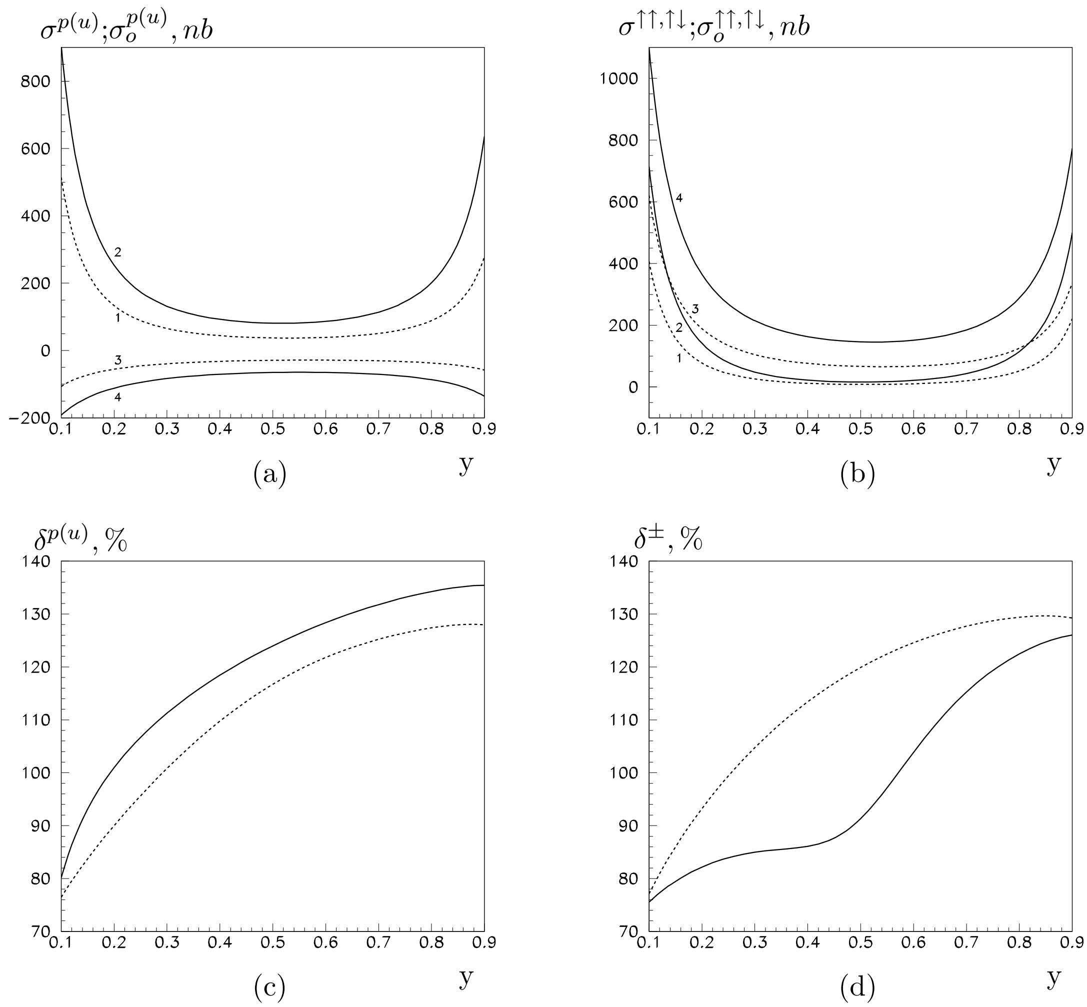


**Figure 3.** *y-dependence of the Møller scattering cross section and RC to it. (a): $\sigma_o^u$ (1), $\sigma^u$ (2), $\sigma_o^p$ (3), $\sigma^p$ (4); (b): $\sigma_o^{\uparrow\uparrow}$ (1), $\sigma^{\uparrow\uparrow}$ (2), $\sigma_o^{\uparrow\downarrow}$ (3), $\sigma^{\uparrow\downarrow}$ (4); (c): $\delta^{p(u)}$ solid(dotted) line; (d) $\delta^{\pm}$ solid(dotted) line. E=100 Gev. Fixed target experiment. Longitudinal polarized beam and target.*

The quantities shown in figure 3 are defined as follows $\sigma^{\uparrow\uparrow,\uparrow\downarrow}=\sigma_o \pm \sigma$ ($P_B=P_T=1$, $P_B=-P_T=1$), and

$$\sigma^{\uparrow\uparrow,\uparrow\downarrow} = \sigma^u \pm \sigma^p = \sigma_o^u(1+\delta^u) \pm \sigma_o^p(1+\delta^p) = \sigma_o^{\uparrow\uparrow,\uparrow\downarrow}(1+\delta^{\pm}). \tag{30}$$

Where $\sigma_o^{\uparrow\uparrow,\uparrow\downarrow} = \sigma_o^u \pm \sigma_o^p$. The values $\delta^{p(u)}$, $\delta^{\pm}$ which obey the relations

$$\delta^{p(u)} = \frac{\sigma^{p(u)}}{\sigma_o^{p(u)}} \tag{31}$$

and

$$\delta^{\pm} = \frac{\sigma_o^u \delta^u \pm \sigma_o^p \delta^p}{\sigma_o^u \pm \sigma_o^{u,p}} \tag{32}$$

are the lowest-order RC to $\sigma_o^{p(u)}$ and $\sigma_o^{\uparrow\uparrow,\uparrow\downarrow}$ respectively.

From figure 3 follows that RC to polarized(unpolarized) cross section $\delta^{p(u)}$ and "observed" cross section $\delta^{\pm}$ are about 100% ¶. We can see also, that $\delta^p \simeq \delta^u \simeq \delta^{\pm}$; the corrections $\delta^{u(p)}$, $\delta^-$ have similar behavior and the difference, observed between the behaviors of $\delta^{+,-}$ points out a non-vanishing(vanishing) contribution of RC to the polarization asymmetry in the region where $\delta^- > \delta^+ (\delta^- \simeq \delta^+)$. A detailed numerical analysis of RC to cross section and polarization asymmetry for polarized Møller scattering, including experimental conditions, will be a subject of separate investigation.

The formulae proposed in this paper have been tested in a wide kinematical range of beam energy (0.1-4000 Gev.) at fixed target experiment. The obtained results may be used for numerical analysis of observables at fixed target set-up (e.g. for Møller polarimeter) and colliders (e.g. $e^-e^-$, $\mu^-\mu^-$). However, for collideres, proposed results can be used only at low energies, when $\sqrt{s} \ll M_{Z_o}$. For a more accurate calculation at colliders, electroweak corrections have to be considered.

## Acknowledgements

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## Appendix A

In this appendix the quantities (see (8)), which are necessary to calculate the Born cross section are shown.

$$U_1 = \frac{1}{2}\Big(S^2 + Q_1^4\Big) + 2m^2(S - 2Q^2 - m^2)$$

$$U_2 = \frac{1}{2}\Big(S^2 + Q^4\Big) + 2m^2(S - 2Q_1^2 - m^2)$$

$$U_3 = S(S - 4m^2)$$

$$P_{1ll} = Q^2\Big[\frac{Q^2 d_s^2}{2\lambda_s} - S\Big]$$

$$P_{2ll} = Q^2\Big[\frac{Q^2 d_s^2}{2\lambda_s} - 2m^2\Big] + \frac{1}{2}\lambda_s - S d_s$$

$$P_{3ll} = -\frac{1}{\lambda_s}\Big[S^4 + 4m^2(-S^2Q^2 + m^2(-S^2 + 2Q^4 + 4m^2Q^2))\Big]$$

¶ Since the RC of the lowest-order to Born cross section are about 100%, for data analysis in a concrete set-up, either RC of the $\alpha^2$-order should be investigated or experimental conditions are required where radiative effects could be reduced due to experimental cuts.

$$P_{1tt}=P_{2tt}=\frac{2m^2Q^4}{\mathcal{K}^2\lambda_s}\Big(4m^2(4m^4Q^2+m^2(Q^4-2SX_o)-SQ^2X_o)+S^2X_o^2\Big) \tag{33}$$
$$P_{3tt}=-\frac{SQ^4}{\mathcal{K}^2\lambda_s}\Big(4m^2(4m^4Q^2+m^2(Q^4-2SX_o)-SQ^2X_o)+S^2X_o^2\Big)$$
$$P_{1\perp\perp}=-2m^2Q^2 \qquad P_{2\perp\perp}=2m^2(2m^2-X_o) \qquad P_{3\perp\perp}=2m^2(-2S+X_o)+X_oQ^2$$
$$P_{1lt}=P_{1tl}=\frac{mQ^4}{\mathcal{K}\lambda_s}\Big(2m^2(2m^2(2m^2-2S-X_o)+S(S-2X_o)-S^2X_o\Big)$$
$$P_{2lt}=P_{2tl}=\frac{mQ^2}{\mathcal{K}\lambda_s}\Big(2m^2Q^2(2m^2(2m^2-2S-X_o)+S(S-2X_o)+$$
$$+\lambda_s(2m^2(Q^2-2m^2)+SX_o)-S^2X_oQ^2)\Big)$$
$$P_{3lt}=P_{3tl}=-\frac{SQ^4}{\mathcal{K}\lambda_s}\Big(4m^2(4m^4Q^2+m^2(Q^4-2SX_o)-SQ^2X_o)+S^2X_o^2\Big)$$
$$P_{1l\perp}=P_{1\perp l}=P_{1t\perp}=P_{1\perp t}=0 \qquad P_{2l\perp}=P_{2\perp l}=P_{2t\perp}=P_{2\perp t}=0$$
$$P_{3l\perp}=P_{3\perp l}=P_{3t\perp}=P_{3\perp t}=0$$

In (33) $Q_1^2{=}X_o-2m^2$, $d_s{=}S+2m^2$.

## Appendix B

Here we present a set of integrals, which are necessary for calculations of the contribution $\sigma_R^F$ (see (22)). This set is complementary to the one that was given in [15, 17].

The integrals have the form

$$\Big[A\Big]=\int\limits_{t_{min}}^{t_{max}}dt\int\limits_{z_2^{min}}^{z_2^{max}}\frac{dz_2}{\sqrt{R_{z_2}}}A. \tag{34}$$

The limits of kinematical variables $t$ and $z_2$ are $t_{max/min}=\dfrac{vS_x+2m^2Q^2\pm v\sqrt{\lambda_Y}}{2\tau}$, $z_2^{max(min)}$ were given first in [18] (See also [21]). $R_{z_2}$ is the Gram determinant.

$$\Big[\frac{t^2}{z_2}\Big]=\frac{1}{2A}\Big[\frac{v}{\tau^2}(ER_{ex}+F\tau)-\frac{3BEv}{A\tau}+\Big(\frac{3B^2}{A}-C\Big)L_A\Big] \tag{35}$$
$$\Big[\frac{t^2}{z_1}\Big]=-\Big[\frac{t^2}{z_2}\Big](S\to -X) \tag{36}$$
$$\Big[\frac{t}{z_2^2}\Big]=\frac{1}{A}\Big[\frac{1}{Mv}\Big(\frac{Q^2SW(Q^2+X)}{m^2}-EF\Big)+EL_A\Big] \tag{37}$$
$$\Big[\frac{t}{z_1^2}\Big]=\Big[\frac{t}{z_2^2}\Big](S\to -X) \tag{38}$$
$$\Big[\frac{t^2}{z_2^2}\Big]=\frac{1}{A^2}\Big[\frac{1}{Mv}\Big(E(AH+2BF)-2WEQ^2Sv-$$
$$-W(X+Q^2)\Big(Fv+\frac{BQ^2S}{m^2}\Big)\Big)+(AF-3BE)L_A\Big] \tag{39}$$

$$\Big[\frac{t^2}{z_1^2}\Big]=\Big[\frac{t^2}{z_2^2}\Big](S\to -X) \tag{40}$$

$$\Big[\frac{1}{z_2(t-R)}\Big]= L_R = \frac{1}{SX}\ln\frac{(t_2-R)[SX+(X+Q^2)(t_1-R)+\sqrt{-C(t_1)}]}{(t_1-R)[SX+(X+Q^2)(t_2-R)+\sqrt{-C(t_2)}]} \tag{41}$$

$t_2=t_{max}, t_1=t_{min}$

$$\Big[\frac{1}{z_2^2(t-R)}\Big]= \frac{1}{S^2X^2}\Big[F_RL_R-\frac{B_R}{m^2v}\Big] \tag{42}$$

$$\Big[\frac{1}{z_2(t-R)^2}\Big]= \frac{1}{S^2X^2}\Big[\frac{vF_R}{X_{12}}-B_RL_R\Big] \tag{43}$$

$$\Big[\frac{1}{z_2^2(t-R)^2}\Big]= \frac{1}{S^2X^2}\Big[L_R(E-\frac{3F_RB_R}{S^2X^2})+\frac{A}{m^2v}+\frac{v}{X_{12}}\Big(\frac{3F_R^2}{S^2X^2}-2\lambda_Y\Big)\Big] \tag{44}$$

$$\Big[\frac{1}{z_2(t-R)^3}\Big]= \frac{1}{2S^2X^2}\Big[\frac{v}{X_{12}}\Big(\frac{\Theta}{X_{12}}-\frac{3F_RB_R}{S^2X^2})+L_R(\frac{3B_R^2}{S^2X^2}-A)\Big] \tag{45}$$

$$\Big[\frac{1}{(z_1-P)}\Big]= L_P = \frac{1}{\hat{A}}\ln\frac{X_ot_2-Sv+\sqrt{-\hat{N}(t_2)}}{X_ot_1-Sv+\sqrt{-\hat{N}(t_1)}} \tag{46}$$

$$\Big[\frac{t}{(z_1-P)}\Big]= \frac{1}{\hat{A}}\Big[\frac{\hat{E}v}{\tau}-\hat{B}_PL_P\Big] \tag{47}$$

$$\Big[\frac{1}{t(z_1-P)}\Big]= L_N = \frac{1}{Sv}\ln\frac{t_2[Sv-X_ot_1+\sqrt{-\hat{N}(t_1)}]}{t_1[Sv-X_ot_2+\sqrt{-\hat{N}(t_2)}]} \tag{48}$$

$$\Big[\frac{1}{u(z_1-P)}\Big]= L_\xi = \frac{1}{XQ^2}\ln\frac{(t_1-S_x)[XQ^2+X_o(t_2-S_x)+\sqrt{-\hat{N}(t_2)}]}{(t_2-S_x)[XQ^2+X_o(t_1-S_x)+\sqrt{-\hat{N}(t_1)}]} \tag{49}$$

$$\Big[\frac{1}{u^2(z_1-P)}\Big]= \frac{1}{X^2Q^4}\Big[\frac{\hat{F}_\xi v}{\xi_{12}}-\hat{B}_\xi L_\xi\Big] \tag{50}$$

$$\Big[\frac{1}{(z_1-P)^2}\Big]= \frac{1}{X_o\hat{N}_{12}}\Big[\hat{F}+X_o(\lambda_Y+\hat{E})\Big] \tag{51}$$

$$\begin{aligned}\Big[\frac{t}{(z_1-P)^2}\Big]= \frac{1}{\hat{A}}\Big[\frac{1}{\hat{N}_{12}}\Big(&(X+v)Wv-X_o(\hat{E}^2+\lambda_Y(\hat{E}-\hat{A}))+\\ &+2\lambda_Y\hat{B}_P-\hat{E}\hat{F}\Big)+\hat{E}L_P\Big]\end{aligned} \tag{52}$$

$$\Big[\frac{1}{u(z_1-P)^2}\Big]= \frac{1}{X^2Q^4}\Big[\frac{1}{\hat{N}_{12}}\Big((X-Q^2)(\lambda_Y\hat{A}+\hat{E}^2-Wv)+\hat{E}\hat{F}_\xi\Big)+\hat{F}_\xi L_\xi\Big] \tag{53}$$

$$\Big[\frac{1}{u^2(z_1-P)^2}\Big]= \frac{1}{X^2Q^4}\Big[\frac{v}{\xi_{12}}\Big(\frac{3\hat{F}_{PR}^2}{S^2X^2}-2\lambda_Y\Big)+\frac{1}{\hat{N}_{12}}\Big(Wv-\hat{A}\lambda_Y\Big)+L_\xi(\hat{E}-\frac{3\hat{B}_\xi\hat{F}_\xi}{X^2Q^4})\Big] \tag{54}$$

$$\Big[\frac{1}{(z_1-P)(t-R)}\Big]= L_{PR} = \frac{1}{SX}\ln\frac{(t_2-R)[SX+X_o(t_1-R)+\sqrt{-\hat{N}(t_1)}]}{(t_1-R)[SX+X_o(t_2-R)+\sqrt{-\hat{N}(t_2)}]} \tag{55}$$

$$\Big[\frac{1}{(z_1-P)(t-R)^2}\Big]= \frac{1}{S^2X^2}\Big[\frac{v\hat{F}_{PR}}{X_{12}}-\hat{B}_{PR}L_{PR}\Big] \tag{56}$$

$$\Big[\frac{1}{(z_1-P)(t-R)^3}\Big]=\frac{1}{2S^2X^2}\Big[\frac{v}{X_{12}}\Big(\frac{\Sigma}{X_{12}}-\frac{3\hat{F}_{PR}\hat{B}_{PR}}{S^2X^2}\Big)+L_{PR}\Big(\frac{3B_{PR}^2}{S^2X^2}-\hat{A}\Big)\Big] \tag{57}$$

$$\Big[\frac{1}{(z_1-P)^2(t-R)}\Big]=\frac{1}{S^2X^2}\Big[\frac{1}{\hat{N}_{12}}\Big(Q^2(\lambda_Y\hat{A}+\hat{E}^2-Wv)-\hat{E}\hat{F}_{PR}\Big)+\hat{F}_{PR}L_{PR}\Big] \tag{58}$$

$$\Big[\frac{1}{(z_1-P)^2(t-R)^2}\Big]=\frac{1}{S^2X^2}\Big[\frac{v}{X_{12}}\Big(\frac{3\hat{F}_{PR}^2}{S^2X^2}-2\lambda_Y\Big)+\frac{1}{\hat{N}_{12}}\Big(Wv-\hat{A}\lambda_Y\Big)+$$
$$+L_{PR}\Big(\hat{E}-\frac{3\hat{B}_{PR}\hat{F}_{PR}}{S^2X^2}\Big)\Big]. \tag{59}$$

$$\begin{array}{ll}
A=(X+Q^2)^2-4m^2\tau & \hat{A}=(S-Q^2)^2-4m^2\tau \\
B=vB_o-Q^2\lambda_s & \hat{B}=v\hat{B}_o-Q^2\lambda_x \\
B_o=SQ^2+2m^2S_x & \hat{B}_o=-XQ^2+2m^2S_x \\
C=Q^2\lambda_s & \hat{C}=Q^2\lambda_x \\
E=vX-Q^2d_s & \hat{E}=vS-Q^2(X-2m^2) \\
F=Q^2F_o & \hat{F}=Q^2\hat{F}_o \\
F_o=SS_x+2m^2Q^2 & \hat{F}_o=XS_x-2m^2Q^2
\end{array} \tag{60}$$

$$\begin{array}{lll}
M=Q^2(SX-Q^2m^2) & H=EQ^4m^2+FR_{ex} & W=M-m^2\lambda_Y \\
\hat{B}_{PR}=SX(X+v) & \hat{H}=\hat{E}Q^4m^2+\hat{F}R_{ex} & R_{ex}=S_xv+2m^2Q^2 \\
B_R=SX(X+Q^2) & F_R=SX(v-Q^2) & \hat{B}_P=-Sv(X+v) \\
\hat{F}_P=-Sv(v+Q^2) & \hat{F}_{PR}=F_R & \hat{B}_\xi=XQ^2(X+v) \\
P=X+v-2m^2 & S_x=S-X & \lambda_Y=(Q^2+v)^2+4m^2Q^2
\end{array} \tag{61}$$

$$\begin{array}{lll}
\hat{F}_\xi=-XQ^2(v+Q^2) & \Sigma=SXv(SQ^2-Xv) & R=S-2m^2 \\
t_1t_2=\dfrac{m^2Q^4}{\tau} & \Theta=SXv(XQ^2-Sv) & X_{12}=SXv \\
\xi_{12}=m^2v^2 & \hat{N}_{12}=5SXQ^2v+2(X^2Q^4+S^2v^2) &
\end{array} \tag{62}$$

$$\begin{aligned}
\sqrt{-C(t_{1(2)})}&=\frac{Et_{1(2)}+F}{\sqrt{\lambda_Y}} \\
\sqrt{-\hat{C}(t_{1(2)})}&=\frac{\hat{E}t_{1(2)}+\hat{F}}{\sqrt{\lambda_Y}} \\
\sqrt{-\hat{N}(t_{1(2)})}&=\sqrt{-\hat{C}(t_{1(2)})}+X_o\sqrt{\lambda_Y}.
\end{aligned} \tag{63}$$

## Appendix C

The quantities A (see (20)) are given by

$$A_1=2(Q^2-2m^2)$$
$$A_2=-2m^2(3S+X_o)$$

$$
\begin{aligned}
A_3 &= 2\Big(8m^4 - 6m^2X_o + X_o^2\Big)\\
A_4 &= 2(2S^2 + X_o^2 - 2m^2(3S + Q^2))\\
A_{5ll} &= \frac{Q^2}{\lambda_s}\Big(4m^4 + 4m^2S + S^2\Big) - 2S - Q^2\\
A_{6ll} &= m^2\Big(1 - \frac{SQ^2}{\lambda_s}\Big) + 2\Big(-\frac{2S^2Q^2}{\lambda_s} + 3Q^2 - 2S\Big) + \\
&\quad + \frac{1}{m^2}\Big(-2\lambda_s - \frac{S^3Q^2}{\lambda_s} + 2S^2 + SQ^2\Big)\\
A_{7ll} &= \frac{2m^4}{\lambda_s}\Big(Q^4 - 2SX_o\Big) - 2m^4 + 2m^2\Big(2S - Q^2 - \frac{SQ^2X_o}{\lambda_s}\Big) + X_o\Big(\frac{S^2X_o}{\lambda_s} - X_o - 2S\Big)\\
A_{8ll} &= -\frac{4m^4Q^2}{\lambda_s}\Big(2S + Q^2\Big) + 2m^2\Big(5Q^2 + 2S - \frac{3SQ^4}{\lambda_s}\Big) + \\
&\quad + \Big(\frac{S^2Q^2}{\lambda_s}(2S - 3Q^2) - 2S(S + Q^2) + 5Q^4\Big) + \frac{SQ^2}{2m^2}\Big(\frac{S^3}{\lambda_s} - \frac{S^2Q^2}{\lambda_s} - S + Q^2\Big)\\
A_{5tt} &= \frac{2m^2}{\mathcal{K}^2\lambda_s}\Big(4m^2(4m^6 + 4m^4Q^2 + m^2(Q^4 - 2SX_o) - SQ^2X_o) + S^2X_o^2\Big)\\
A_{6tt} &= -\frac{2Q^2S}{\mathcal{K}^2\lambda_s}\Big(4m^2(4m^6 + 4m^4Q^2 + m^2(Q^4 - 2SX_o) - SQ^2X_o) + S^2X_o^2\Big)\\
A_{7tt} &= \frac{2m^2Q^4}{\mathcal{K}^2\lambda_s}\Big(4m^2(4m^6 + 4m^4Q^2 + m^2(Q^4 - 2SX_o) - SQ^2X_o) + S^2X_o^2\Big)\\
A_{8tt} &= \frac{2m^2Q^4}{\mathcal{K}^2\lambda_s}\Big(4m^2(12m^6Q^2 + 2m^4(-2SX_o + 3Q^2) + m^2Q^2(Q^4 - 4SX_o)) + \\
&\qquad + 2m^2S(X_o(S - 2Q^2)(S + Q^2)) + Q^2S^2X_o^2\Big).\\
A_{5\perp\perp} &= -8mQ^2(m^2 + Q^2)\\
A_{6\perp\perp} &= 2mX_o(X_o - 8m^2)\\
A_{7\perp\perp} &= 4m(-2m^2(S + 2Q^2) + S(S + Q^2) - 2Q^4)\\
A_{8\perp\perp} &= 4mQ^2(3S - 2Q^2 - 4m^2).
\end{aligned} \tag{64}
$$

All details about the calculation of the quantities given above can be found at htpp://www.hep.by/mollerad.htm

## Appendix D

In this appendix we present the explicit formulae for the two-photon exchange contribution (see (21)). The corrections $\delta_{2\gamma(BT)}^{h,u(p)}$ have the form

$$
\begin{aligned}
\delta_{2\gamma}^{l,u} = \; & -\frac{Q^2}{U_1}\Big[\frac{S}{2Q^2}\Big(U_1 + S^2\Big)K_s + \frac{X_o}{2Q^2}\Big(U_1 + X_o^2\Big)K_{x_o} - \\
& - \frac{1}{4Q_1^2}\Big(Q^2Q_1^4 + X_o\lambda_{x_o}\Big)L_{x_o} + \frac{1}{4d_s}\Big(Q^2d_s^2 - S\lambda_s\Big)L_s - \\
& - (S + X_o)\Big(4Q_m^2G_o^m + 8m^2g_1^m + \frac{1}{2}\ln\frac{m^2}{Q^2}\Big)\Big]
\end{aligned} \tag{65}
$$

$$\Re = \Bigl(\frac{\lambda_s Q^2}{4\Delta} - S\Bigr)K_s - \Bigl(\frac{\lambda_{x_o} Q^2}{4\Delta} + X_o\Bigr)K_{x_o} - \frac{2Q^4(S+X_o)}{\Delta}G_o^m$$

$$\begin{aligned} \delta_{2\gamma}^{i1,u} = \quad & \Re - \frac{Q^2 m^2}{U_3}\Bigl[S^2((S+Q^2)\hat{a}_1 - X_o a_1) + S^2(\hat{b}_o + b_o) + 4S^2 b_o - \\ & -2m^2 Q^2(\hat{b}_1 - b_1) + m^2(S+X_o)(\hat{b}_4 + b_4) + 2m^2(S+Q^2)\hat{b}_4\Bigr] \end{aligned} \tag{66}$$

$$\begin{aligned} \delta_{2\gamma ll}^{l,p} = \quad & \Re + \frac{2Q^2 m^2}{P_{1ll}}\Bigl[Q^2(S+X_o)(S\hat{a}_1 + X_o a_1) + \\ & +(S^2 + X_o^2)(\hat{b}_o + b_o) + S(X_o\hat{b}_4 - S b_4)\Bigr] \end{aligned} \tag{67}$$

$$\begin{aligned} \delta_{2\gamma tt}^{l,p} = \quad & \Re + \frac{Q^2 m^2}{P_{1tt}}\Bigl[X_o Q^2((-3S + 2Q^2)\hat{a}_1 - 4S a_1) + \\ & +4Q^2 X_o(\hat{b}_1 - b_1) + 6X_o Q^2(\hat{b}_4 - b_4)\Bigr] \end{aligned} \tag{68}$$

$$\begin{aligned} \delta_{2\gamma\perp\perp}^{l,p} = \quad & \Re + \frac{Q^2 m^4}{P_{1\perp\perp}}\Bigl[X_o(S\hat{a}_1 + Q^2 a_1) - 2S(\hat{b}_o + b_o) + \\ & +(\frac{3}{4}Q^2 - S)(\hat{b}_1 - b_1) - \frac{1}{2}X_o(\hat{b}_4 - b_4)\Bigr] \end{aligned} \tag{69}$$

$$\delta_{2\gamma ll}^{i1,p} = \quad \Re + \frac{8Q^2 S^2 m^2}{P_{3ll}}\Bigl[S\hat{a}_1 + X_o a_1 + \hat{b}_o + b_o - b_4\Bigr] \tag{70}$$

$$\delta_{2\gamma tt}^{i1,p} = \quad \Re + \frac{Q^2 m^2}{P_{3tt}}\Bigl[Q^4(2S\hat{a}_1 + X_o a_1) + S X_o(\hat{b}_1 + b_1) - 3SQ^2 b_4\Bigr] \tag{71}$$

$$\begin{aligned} \delta_{2\gamma\perp\perp}^{i1,p} = \quad & \Re + \frac{Q^2 m^4}{P_{3\perp\perp}}\Bigl[X_o^2\hat{a}_1 + (S+Q^2)a_1 + 12X_o(\hat{b}_o + b_o) + \\ & +S(\hat{b}_1 + b_1) - 2Q^2(\hat{b}_4 + b_4)\Bigr]. \end{aligned} \tag{72}$$

The rest ones are obtained from the corrections given above, using cross-symmetry:

$$\begin{array}{ll} \delta_{2\gamma}^{e,u} = \delta_{2\gamma}^{l,u}(\Im_1) & \delta_{2\gamma}^{i2,u} = \delta_{2\gamma}^{i1,u}(\Im_2) \\ \delta_{2\gamma ll}^{e,p} = \delta_{2\gamma ll}^{l,p}(P_{1ll} \to P_{2ll},\ \Im_2) & \delta_{2\gamma ll}^{i2,p} = \delta_{2\gamma ll}^{i1,p}(\Im_2) \\ \delta_{2\gamma tt}^{e,p} = \delta_{2\gamma tt}^{l,p}(P_{1tt} \to P_{2tt},\ \Im_2) & \delta_{2\gamma tt}^{i2,p} = \delta_{2\gamma tt}^{i1,p}(\Im_2) \\ \delta_{2\gamma\perp\perp}^{e,p} = \delta_{2\gamma\perp\perp}^{l,p}(P_{1\perp\perp} \to P_{2\perp\perp},\ \Im_2) & \delta_{2\gamma\perp\perp}^{i2,p} = \delta_{2\gamma\perp\perp}^{i1,p}(\Im_2). \end{array} \tag{73}$$

Here $\Im_1$ are the following replacements:

$$U_1 \to U_2,\ Q^2 \to Q_1^2,\ X_o \to Q_m^2,\ \lambda_{x_o} \to \lambda_m,\ L_{x_o} \to L_m,$$

$$K_s \to \tilde{K}_s,\ K_{x_o} \to \tilde{K}_{x_o},\ G_o \to \tilde{G}_o,\ g_1^m \to \tilde{g}_1^m$$

and $\Im_2$ are the replacements:

$$\Delta \to \tilde{\Delta},\ Q^2 \to Q_1^2,\ X_o \to Q_m^2,\ \lambda_{x_o} \to \lambda_m,\ G_o \to \tilde{G}_o,$$

$$K_s \to \tilde{K}_s,\ K_{x_o} \to \tilde{K}_{x_o},\ \hat{a}_1 \to \hat{a}_{11},\ a_1 \to a_{11},\ \hat{b}_o \to \hat{b}_{o1},$$

$$b_o \to b_{o1},\ b_1 \to b_{11},\ \hat{b}_1 \to \hat{b}_{11},\ b_4 \to b_{41},\ \hat{b}_4 \to \hat{b}_{41}.$$

The explicit form of $K_s$, $K_{x_o}$, $G_o^m$ and $g_1^m$ was given in [20] by (19). For $\tilde{K}_s$, $\tilde{K}_{x_o}$, $\tilde{G}_o^m$ and $\tilde{g}_1^m$ we have

$$\begin{aligned}\tilde{K}_s = &-L_s \ln\frac{Q_1^2}{S-2m^2}-\\ &-\frac{2}{\sqrt{\lambda_s}}\Big[Li_2\Big(\frac{2m^2}{S+\sqrt{\lambda_s}}\Big)-\frac{2}{\sqrt{\lambda_s}}\Big[Li_2\Big(\frac{S+\sqrt{\lambda_s}}{2m^2}\Big)+\pi^2\Big]\end{aligned} \tag{74}$$

$$\begin{aligned}\tilde{K}_{x_o} = &L_m \ln\frac{Q_1^2 Q^2}{\lambda_m}+\\ &+\frac{2}{\sqrt{\lambda_m}}\Big[Li_2\Big(-\frac{4m^2Q^2}{(Q^2+\sqrt{\lambda_m})^2}\Big)-Li_2\Big(\frac{(Q^2+\sqrt{\lambda_m})^2}{4m^2Q^2}\Big)\Big]\end{aligned} \tag{75}$$

$$\begin{aligned}\tilde{G}_o^m = &-\frac{1}{4}\Big(L_{mex}\ln\frac{|Q_1^2|}{m^2}+\\ &+\frac{1}{\sqrt{\lambda_{mex}}}\Big[Li_2\Big(-\frac{Q_1^2+\sqrt{\lambda_{mex}}}{2m^2}\Big)-Li_2\Big(\frac{2Q_1^2}{Q_1^2+\sqrt{\lambda_{mex}}}\Big)+\pi^2\Big]\Big)\end{aligned} \tag{76}$$

$$\tilde{g}_1^m = \frac{1}{Q_1^2+4m^2}\Big(\frac{1}{2}\ln\frac{|Q_1^2|}{m^2}+Q_1^2\tilde{G}_0^m\Big). \tag{77}$$

The coefficients that appear in (65)-(73) are:

$$\Delta = \tfrac{1}{2}Q_1^2 d_s \qquad \tilde{\Delta} = \tfrac{1}{2}Q^2 d_s$$

$$\begin{aligned}
a_1 &= -\frac{1}{\Delta}\Big[(K_{x_o}+8G_o^m)(X_o+2m^2)\Big] & \hat{a}_1 &= \frac{1}{\Delta}\Big[(K_s+8G_o^m)(S-2m^2)\Big]\\
a_{11} &= -\frac{1}{\tilde{\Delta}}\Big[(\tilde{K}_{x_o}+8\tilde{G}_o^m)Q^2\Big] & \hat{a}_{11} &= \frac{1}{\tilde{\Delta}}\Big[(\tilde{K}_s+8\tilde{G}_o^m)(S-2m_e^2)\Big]\\
b_0 &= -\frac{1}{16}\Big[K_{x_o}+8Q^2a_1\Big] & \hat{b}_0 &= -\frac{1}{16}\Big[K_s+8Q^2\hat{a}_1\Big]\\
b_{01} &= -\frac{1}{16}\Big[\tilde{K}_{x_o}+8Q_1^2a_{11}\Big] & \hat{b}_{01} &= -\frac{1}{16}\Big[\tilde{K}_s+8Q_1^2\hat{a}_{11}\Big]\\
R_1 &= -\frac{1}{4}L_{x_o}-2g_1^m+Q^2a_1 & \hat{R}_1 &= -\frac{1}{4}L_s-2g_1^m+Q^2\hat{a}_1\\
R_{11} &= -\frac{1}{4}L_m-2\tilde{g}_1^m+Q_1^2a_{11} & \hat{R}_{11} &= -\frac{1}{4}L_s-2\tilde{g}_1^m+Q_1^2\hat{a}_{11}
\end{aligned} \tag{78}$$

$$b_1 = \frac{1}{\Delta}\Big[Q_1^2R_1+(g_1^m-2b_0)(Q^2+4m^2)\Big]$$

$$
\begin{aligned}
\hat{b}_1 &= \frac{1}{\Delta}\Big[-d_s\hat{R}_1 + (g_1^m - 2\hat{b}_0)(Q^2+4m^2)\Big] \\
b_{11} &= \frac{1}{\tilde{\Delta}}\Big[Q^2R_{11} + (\tilde{g}_1^m - 2b_{01})(Q_1^2+4m^2)\Big] \\
\hat{b}_{11} &= \frac{1}{\tilde{\Delta}}\Big[-d_s\hat{R}_{11} + (\tilde{g}_1^m - 2\hat{b}_{01})(Q_1^2+4m^2)\Big] \\
b_4 &= \frac{1}{\Delta}\Big[Q_1^2R_1 + (g_1^m - 2b_0)(Q^2+2X_o)\Big] \\
\hat{b}_4 &= \frac{1}{\Delta}\Big[-d_s\hat{R}_1 + (g_1^m - 2\hat{b}_0)(2S-Q^2)\Big] \\
b_{41} &= \frac{1}{\tilde{\Delta}}\Big[Q^2R_{11} + (\tilde{g}_1^m - 2b_{01})(Q_1^2+2Q_m^2)\Big] \\
\hat{b}_{41} &= \frac{1}{\tilde{\Delta}}\Big[-d_s\hat{R}_{11} + (\tilde{g}_1^m - 2\hat{b}_{01})(2S-Q_1^2)\Big].
\end{aligned}
\tag{79}
$$

## Appendix E

Here we give schematically the formulae of the cross section $\sigma_R^F$ (see (22)). This contribution was found by means of the system of analytical calculations REDUCE [27]. The calculations were carried out using the scheme given in [17] (see also [20]). We have

$$
\sigma_R^F = \frac{\alpha^3 S}{\lambda_s}\Big[\sum_{k=l,e,i}\sigma^{k,u} + \sum_B\sum_T P_B P_T \sum_{k=l,e,i}\sigma_{BT}^{k,p}\Big] \tag{80}
$$

where

$$
\begin{aligned}
\sigma^{l,u} &= \frac{1}{Q^4}S_1^{l,u} + S_2^{l,u} + \frac{1}{Q^2}S_3^{l,u} \\
\sigma^{e,u} &= \frac{1}{t_m^2}S_1^{e,u} + S_2^{e,u} + \frac{1}{t_m}S_3^{e,u} \\
\sigma^{i,u} &= \frac{1}{t_m}S_1^{i,u} + \frac{1}{t_mQ^2}S_2^{i,u} + S_3^{i,u} + \frac{1}{Q^2}S_4^{i,u} \\
\sigma_{BT}^{l,p} &= \frac{1}{Q^4}S_{1BT}^{l,p} + S_{2BT}^{l,p} + \frac{1}{Q^2}S_{3BT}^{l,p} \\
\sigma_{BT}^{e,p} &= \frac{1}{t_m^2}S_{1BT}^{e,p} + S_{2BT}^{e,p} + \frac{1}{t_m}S_{3BT}^{e,p} \\
\sigma_{BT}^{i,p} &= \frac{1}{t_m}S_{1BT}^{i,p} + \frac{1}{t_mQ^2}S_{2BT}^{i,p} + S_{3BT}^{i,p} + \frac{1}{Q^2}S_{4BT}^{i,p}
\end{aligned}
\tag{81}
$$

where $t_m = X - 2m^2$. The results of calculation of $S_f^{k,u}$ ($f$=1, 2, 3, 4) and $S_{fBT}^{k,p}$ ($f$=1, 2, 3, 4) were obtained as a set of output REDUCE files . Because the obtained exact result is very cumbersome we do not present him here. The whole set of REDUCE files, necessary for calculations, can be found at the http://www.hep.by/mollerad.htm

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